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The slowly rotating near extremal D1-D5 system as a 'hot tube'

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Abstract

The geometry of the D1-D5 system with a small angular momentum j has a long throat ending in a conical defect. We solve the scalar wave equation for low energy quanta in this geometry. The quantum is found to reflect off the end of the throat, and stay trapped in the throat for a long time. The length of the throat for j=1/2 equals $n_1 n_5 R$, the length of the effective string in the CFT; we also find that at this distance the incident wave becomes nonlinear. Filling the throat with several quanta gives a 'hot tube' which has emission properties similar to those of the near extremal black hole.

1 Introduction.

String theory has had considerable success in explaining the quantum properties of black holes. We do not however have a clear understanding of how the information in infalling matter transfers itself to the emerging Hawking radiation, so we do not have a resolution of the 'information paradox'. It is thus interesting to probe the physics of systems that are close to the threshold of black hole formation, with a view towards uncovering the mechanism of information transfer.

The D1-D5 system has been central to our understanding of black holes through string theory. The D1-D5-momentum bound state yields a microscopic count of states that agrees exactly with the Bekenstein entropy for extremal [1] and near-extremal [2] black holes. This system also emits, through a unitary process, low energy radiation that agrees exactly in spin dependence and radiation rate with the Hawking radiation expected from the hole [3]. In the limit where we take the momentum charge to be much less than the other two charges one finds that the above agreement persists at higher orders in the energy of the infalling quantum, and thus emission from the microscopic system reproduces the grey-body factors of Hawking radiation [4].

In this paper we look at the D1-D5 system with a small amount of angular momentum. The geometry in the absence of rotation exhibits an S^3 that asymptotes to a nonzero size as the radial coordinate r tends to zero. The distance to $r = 0$ is infinite. We will call this region where the size of the S^3 approximates its asymptotic value as the 'throat' of the geometry. It was shown in [5] that the presence of rotation causes the throat to truncate at some distance in a conical defect. For small values of rotation the throat is long. We will refer to this truncated throat as a 'tube'.

With this system we perform the following investigations:

(a) We consider the wave equation for a massless minimally coupled scalar in such a geometry. It is known that the radial equation separates from the angular equation. Performing this separation we solve the radial equation for long wavelength modes by joining the solution in the large r region to the solution in the small r region. It turns out that the wave reflects naturally off the end of the tube, and thus we do not have to choose between different possible boundary conditions at $r = 0$.

Long wavelength modes incident from infinity have a small probability for entering the throat, and the wave reflected from the end of the throat has a similar small probability for leaving the throat and escaping back to infinity. We compute the 'time delay' from the wavefunction to find how long the quantum stays in the 'tube'. We find time delays corresponding to traveling $1, 2, 3, \ldots$ times down the tube before emerging. These time delays agree with the travel time estimated from the path of a null geodesic bouncing off the ends of the tube.

(b) Let R be the radius of the direction which is common to the D1 and D5 branes, and n_1, n_5 be the number of D1 and D5 branes. It is known that the length scale $n_1 n_5 R$ appears in the physics of the D1-D5 black hole. Thermodynamically, it is the wavelength of the last quantum emitted in a thermal approximation, and microscopically it is the length of the 'effective string' formed in the D1-D5 bound state. In solving the wave equation mentioned above we find that this length appears as a feature of the geometry, in several different ways, as follows.

Looking at a class of ground states of the D1-D5 system we observe that the minimal value of angular momentum for at least this class of states is $j = 1/2$ rather than $j = 0$. Thus we look at the geometry for this minimal value of rotation. We find that in this case the time needed for a quantum to bounce off the end of the tube is $\sim Rn_1n_5$.

Next, we consider the geometry as having no rotation, but throw in a quantum that carries $j \sim 1/2$. We ask if the back-reaction of this quantum will cause horizon formation (due to its energy) or truncate the throat in a conical defect (due to its angular momentum). We find that the former happens if the wavelength is smaller than $\sim Rn_1n_5$, while the latter happens if the wavelength exceeds $\sim Rn_1n_5$.

We then look at the waveform for a spinless quantum propagating in the geometry with no rotation. We observe that the amplitude of the wave increases as $\sim r^{-1/2}$ as $r \to 0$. We estimate the point where the wave amplitude becomes large enough to turn on nonlinear effects. This point turns out to be a distance $\sim Rn_1n_5$ along the throat.

(c) Since quanta have a small probability to exit from the tube, we can trap them for a long time in the tube and make a thermal bath. We refer to this as a 'hot tube'. We investigate the properties of such a thermal bath, and observe that it radiates in a manner similar to a near extremal black hole.

We conclude the paper with some speculations about the quantum nature of the geometry deep down the throat, and implications for the information problem of black holes.

The plan of this paper is as follows. Sections 2, 3 and 4 study the wave equation in the rotating geometry. Section 5 discusses the length scale Rn_1n_5. In section 6 we look at the thermal properties of the hot tube. Section 7 examines more general possibilities like 'branching throats'. Section 8 is a discussion where we comment on the information paradox.

2 Wave equation for the scalar field.

We begin with the metric for the rotating D1–D5 system [6, 5]:

$$\begin{aligned} ds^2 &= -\frac{1}{h}(dt^2 - dy^2) + hf\left(d\theta^2 + \frac{dr^2}{r^2+a^2}\right) - \frac{2a\sqrt{Q_1Q_5}}{hf}\left(\cos^2\theta dy d\psi + \sin^2\theta dt d\phi\right) \\ &+ h\left[\left(r^2 + \frac{a^2 Q_1 Q_5 \cos^2\theta}{h^2 f^2}\right)\cos^2\theta d\psi^2 + \left(r^2 + a^2 - \frac{a^2 Q_1 Q_5 \sin^2\theta}{h^2 f^2}\right)\sin^2\theta d\phi^2\right], \end{aligned} \tag{2.1}$$

where

$$f = r^2 + a^2 \cos^2\theta, \qquad h = \left[\left(1 + \frac{Q_1}{f}\right)\left(1 + \frac{Q_5}{f}\right)\right]^{1/2} \tag{2.2}$$

The parameter a can be written in terms of the dimensionless variable $\gamma = \frac{2J}{n_1 n_5}$:

$$a = \frac{\sqrt{Q_1 Q_5}}{R}\gamma \tag{2.3}$$

Here J is the angular momentum in units of $\hbar$; thus it is an integer or half integer. The maximum value of J is $\frac{1}{2}n_1 n_5$, so the maximum value of γ is unity.

Let us consider a minimally coupled scalar in this background. The wave equation for such a scalar reads:

$$\Box\Phi + M^2\Phi = 0. \tag{2.4}$$

where:

$$\Box\Phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right) \tag{2.5}$$

We can naturally decompose the wave operator into three parts:

$$\Box\Phi \equiv \Box_{r,\theta}\Phi + \Box_{t,\phi}\Phi + \Box_{y,\psi}\Phi, \tag{2.6}$$

where each component contains only derivatives with respect to variables listed in the subscript. For example,

$$\begin{aligned}\Box_{t,\phi}\Phi \equiv \frac{1}{\sqrt{-g}}\Big[&\partial_t\left(\sqrt{-g}g^{tt}\partial_t\Phi\right) + \partial_t\left(\sqrt{-g}g^{t\phi}\partial_\phi\Phi\right) + \partial_\phi\left(\sqrt{-g}g^{t\phi}\partial_t\Phi\right)\\ &+\partial_\phi\left(\sqrt{-g}g^{\phi\phi}\partial_\phi\Phi\right)\Big]\end{aligned} \tag{2.7}$$

The metric (2.1) has four Killing vectors corresponding to translation along t, ϕ, ψ and y coordinates, so we can look for the solution in the following form:

$$\Phi(t, r, \theta, \phi, \psi, y) = \exp(-i\tilde{\omega}t + im\phi + in\psi + i\tilde{\lambda}y)\tilde{\Phi}(r, \theta). \tag{2.8}$$

Let us evaluate the three parts of (2.6) separately:

$$\Box_{r,\theta}\Phi = \frac{1}{hf}\left\{\frac{1}{r}\partial_r\left(r(r^2+a^2)\partial_r\Phi\right) + \frac{1}{\sin 2\theta}\partial_\theta\left(\sin 2\theta\partial_\theta\Phi\right)\right\} \tag{2.9}$$

$$\begin{aligned}\Box_{t,\phi}\Phi = \frac{1}{hf}\Big\{&\tilde{\omega}^2R^2\left[\frac{r^2}{R^2} + \frac{Q_1+Q_5}{R^2} + \frac{1}{r^2+a^2}\frac{Q_1Q_5}{R^2}\right] - \sqrt{Q_1Q_5}\frac{2m\tilde{\omega}a}{r^2+a^2} + \frac{m^2a^2}{r^2+a^2}\\ &+\tilde{\omega}^2a^2\cos^2\theta - \frac{m^2}{\sin^2\theta}\Big\}\Phi\end{aligned} \tag{2.10}$$

$$\begin{aligned}\Box_{y,\psi}\Phi = -\frac{1}{hf}\Big\{&\tilde{\lambda}^2R^2\left[\frac{r^2}{R^2} + \frac{Q_1+Q_5}{R^2} + \frac{1}{r^2}\frac{Q_1Q_5}{R^2}\right] + \sqrt{Q_1Q_5}\frac{2n\tilde{\lambda}a}{r^2} + \frac{n^2a^2}{r^2}\\ &+\tilde{\lambda}^2a^2\cos^2\theta + \frac{n^2}{\cos^2\theta}\Big\}\Phi\end{aligned} \tag{2.11}$$

We can see that for general value of the mass M the variables r and θ remain coupled in (2.4), but for massless particles we have a separation. If we consider

$$\tilde{\Phi}(r,\theta) = H(r)\Theta(\theta), \tag{2.12}$$

then the Klein–Gordon equation (2.4) for the massless case:

$$\Box\Phi = 0 \tag{2.13}$$

can be rewritten as a system:

$$\begin{aligned}
&\frac{1}{r}\frac{d}{dr}\left(r(r^2+a^2)\frac{dH}{dr}\right) + \Bigg\{\tilde{\omega}^2 R^2\left[\frac{r^2}{R^2} + \frac{Q_1+Q_5}{R^2} + \frac{1}{r^2+a^2}\frac{Q_1Q_5}{R^2}\right] \\
&\quad -\sqrt{Q_1Q_5}\frac{2m\tilde{\omega}a}{r^2+a^2} + \frac{m^2a^2}{r^2+a^2} - \sqrt{Q_1Q_5}\frac{2n\tilde{\lambda}a}{r^2} - \frac{n^2a^2}{r^2} \\
&\quad -\tilde{\lambda}^2R^2\left[\frac{r^2}{R^2} + \frac{Q_1+Q_5}{R^2} + \frac{1}{r^2}\frac{Q_1Q_5}{R^2}\right]\Bigg\} H - \Lambda H = 0
\end{aligned} \tag{2.14}$$

$$\frac{1}{\sin 2\theta}\frac{d}{d\theta}\left(\sin 2\theta\frac{d\Theta}{d\theta}\right) + \left\{a^2(\tilde{\omega}^2 - \tilde{\lambda}^2)\cos^2\theta - \frac{m^2}{\sin^2\theta} - \frac{n^2}{\cos^2\theta}\right\}\Theta = -\Lambda\Theta \tag{2.15}$$

The angular equation reduces to the Laplacian on S^3 if $a\tilde{\omega}, a\tilde{\lambda} \to 0$, which allows us to determine the approximate eigenvalues:

$$\Lambda = l(l+2) + O((a\tilde{\omega})^2) + O((a\tilde{\lambda})^2) \tag{2.16}$$

where l is a nonnegative integer. We work out the value of Λ to order a^2 in Appendix A, but we note here that we will need only the leading term (2.16). The existence of a separation of variables for such a wave equation was shown in [7].

Let us simplify the radial equation. We introduce a new coordinate

$$x = \frac{r^2R^2}{Q_1Q_5} \tag{2.17}$$

and dimensionless parameters $\omega = R\tilde{\omega}$, $\lambda = R\tilde{\lambda}$. Then the radial equation (2.14) becomes

$$\begin{aligned}
&4\frac{d}{dx}\left(x(x+\gamma^2)\frac{dH}{dx}\right) + \Bigg\{(\omega^2-\lambda^2)\left[\frac{Q_1Q_5}{R^4}x + \frac{Q_1+Q_5}{R^2}\right] \\
&\quad + \frac{(\omega - m\gamma)^2}{x+\gamma^2} - \frac{(\lambda+n\gamma)^2}{x}\Bigg\} H - \Lambda H = 0
\end{aligned} \tag{2.18}$$

From now on we will restrict ourselves to the case $\lambda = 0$; i.e., we will not allow any momentum along the compact direction y.

3 Solution of the wave equation.

3.1 The matching technique

While we can not solve the complete radial equation (2.18), it is possible to solve it in two asymptotic regions and match solutions in an intermediate region for low frequencies ω. This technique has been used several times in the past to study absorption into black holes [8, 9, 3, 4, 10].

3.2 Solution in the outer region.

In the outer region (x large), the equation (2.18) becomes:

$$4x^2H'' + 8xH' + \omega^2 \left\{ \frac{Q_1Q_5}{R^4}x + \frac{Q_1+Q_5}{R^2} \right\} H - \Lambda H = 0 \tag{3.1}$$

The general solution of this equation can be written in terms of Bessel's functions:

$$H_{out}(x) = \frac{1}{\sqrt{x}} \left[C_1 J_\nu \left(\sqrt{\frac{Q_1Q_5\omega^2 x}{R^4}} \right) + C_2 J_{-\nu} \left(\sqrt{\frac{Q_1Q_5\omega^2 x}{R^4}} \right) \right], \tag{3.2}$$

where

$$\nu = \left(1 + \Lambda - \omega^2 \frac{Q_1+Q_5}{R^2} \right)^{1/2} \equiv l + 1 + \epsilon \tag{3.3}$$

and

$$\epsilon \approx -\frac{\omega^2}{l+1} \frac{Q_1+Q_5}{2R^2} \tag{3.4}$$

We have chosen to define the parameter ϵ here for the following reason. We have dropped a term proportional to $\frac{\omega^2}{x}H$ in writing the wave equation in the outer region. However when using the matching technique, we will use this solution in a region where x is small (though $\frac{\omega}{\sqrt{x}} \ll 1$). The value of ϵ given in (3.4) arises from retaining the term proportional to $\omega^2 H$, which is smaller than the term we drop in the matching region. In fact none of the terms containing ω^2 are significant in the matching region, and we could have dropped all of them. Keeping one of the terms does not improve the accuracy of the matching, but allows us to use the basis of two Bessel functions J_ν and $J_{-\nu}$ since ν is shifted away from an integer by the amount ϵ. (We find this trick more convenient than using a basis of J and N functions.) Thus ϵ is a regulator which allows us to use a nearly degenerate basis of functions, and we will see explicitly that ϵ cancels at the end of our computations.

Using the series expansion for the Bessel's functions:

$$J_\mu(z) = \left(\frac{z}{2}\right)^\mu \sum_{n=0}^{\infty} \frac{1}{\Gamma(\mu+n+1)n!} \left[-\left(\frac{z}{2}\right)^2 \right]^n, \tag{3.5}$$

we can express $J_{-\nu}(z)$ in the following form:

$$
\begin{aligned}
J_{-\nu}(z) &= \left(\frac{z}{2}\right)^{-\nu}\left[\sum_{n=0}^{l}\frac{(-z^2/4)^n}{\Gamma(-l-\epsilon+n)n!}+\sum_{n=l+1}^{\infty}\frac{(-z^2/4)^n}{\Gamma(-l-\epsilon+n)n!}\right] \\
&= \left(\frac{z}{2}\right)^{-l-1}\left[-(-1)^l\epsilon l!\left(1+O(z^2)\right)+\frac{1}{(l+1)!}\left(-\frac{z^2}{4}\right)^{l+1}\left(1+O(z^2)+O(\epsilon)\right)\right]
\end{aligned}
\tag{3.6}
$$

Here we have used the fact that $\Gamma(-l-\epsilon)=-\left[(-1)^l\epsilon l!\right]^{-1}$.

We will make an expansion of (3.2) in the region where

$$
\frac{z^2}{4}\equiv\frac{Q_1Q_5\omega^2x}{4R^4}\ll 1. \tag{3.7}
$$

We also will use the fact that $\epsilon \ll 1$. However we will not make any assumptions about a magnitude of the ratio

$$
\frac{z^{2l+2}}{\epsilon} \tag{3.8}
$$

and we will keep both leading terms in (3.6). For the Bessel function $J_\nu(z)$ we will keep only the leading term corresponding to $n=0$ in the expansion (3.5). Then for (3.2) we get:

$$
H_{out}(x)\approx\frac{1}{\sqrt{x}}\left(\frac{Q_1Q_5\omega^2x}{4R^4}\right)^{-\frac{l+1}{2}}\left[-(-1)^lC_2\epsilon l!+\frac{C_1-(-1)^lC_2}{(l+1)!}\left(\frac{Q_1Q_5\omega^2x}{4R^4}\right)^{l+1}\right]. \tag{3.9}
$$

3.3 Solution in the inner region.

Let us now look at the inner region where

$$
x\ll\frac{(Q_1+Q_5)R^2}{Q_1Q_5} \tag{3.10}
$$

Then (2.18) becomes:

$$
4\frac{d}{dx}\left(x(x+\gamma^2)\frac{dH}{dx}\right)+\left\{\omega^2\left[\frac{Q_1+Q_5}{R^2}\right]+\frac{(\omega-m\gamma)^2}{x+\gamma^2}-\frac{(n\gamma)^2}{x}\right\}H-\Lambda H=0 \tag{3.11}
$$

We write $H(x)$ as:

$$
H_{in}(x)=x^\alpha(\gamma^2+x)^\beta G(x), \tag{3.12}
$$

where

$$
\alpha=\frac{n}{2},\qquad \beta=\frac{\omega-\gamma m}{2\gamma} \tag{3.13}
$$

Then the radial equation becomes

$$
\begin{aligned}
&4x(x+\gamma^2)G'' + 4\left[2x(\alpha+\beta+1)+\gamma^2(1+2\alpha)\right]G' \\
&+\left\{4(\alpha+\beta)(\alpha+\beta+1)+\omega^2\frac{Q_1+Q_5}{R^2}-\Lambda\right\}G=0
\end{aligned}
\tag{3.14}
$$

This is the hypergeometric equation. There is a regular and a singular solution at $x = 0$. We choose the regular solution for the following reason. We have a conical defect singularity only where $f = r^2 + a^2\cos^2\theta = 0$, which requires $r = 0, \theta = \frac{\pi}{2}$. But for $r = 0$ and $\theta \neq \frac{\pi}{2}$ we have no singularity. While we cannot assume that the solution is regular at a geometric singularity like a conical defect, we need the solution to be regular at $r = 0, \theta \neq \frac{\pi}{2}$. Since the solution factorizes in the r, θ variables, we find that the function of $x \sim r^2$ must be regular at $x = 0$ to ensure such regularity. We will comment on this issue further at the end of this section.

The solution of (3.14) regular at $x = 0$ reads

$$
G(x) = F\left(p,q;1+2\alpha;-\frac{x}{\gamma^2}\right), \tag{3.15}
$$

$$
p = \frac{1}{2}+\alpha+\beta+\frac{1}{2}\sqrt{1+\Lambda-\omega^2\frac{Q_1+Q_5}{R^2}} \tag{3.16}
$$

$$
q = \frac{1}{2}+\alpha+\beta-\frac{1}{2}\sqrt{1+\Lambda-\omega^2\frac{Q_1+Q_5}{R^2}} \tag{3.17}
$$

Note that we have already encountered the square root appearing in this expression in (3.3) where it was called ν. We had introduced the parameter $\epsilon = \nu - l - 1$. and noted that it served as a regulator allowing us to use a nearly degenerate basis of Bessel functions. We have a similar situation here, and again the precise value of the square root in the above relations is not significant since it would be significantly affected by terms that have been dropped. Again we note that all terms proportional to ω^2 can be dropped in the region of matching. We write

$$
\begin{aligned}
p &= \frac{1}{2}+\alpha+\beta+\frac{1}{2}(l+1+\epsilon') \\
q &= \frac{1}{2}+\alpha+\beta-\frac{1}{2}(l+1+\epsilon')
\end{aligned}
\tag{3.18}
$$

where we have chosen a different symbol ϵ' to denote the regulator allowing us to use a nearly degenerate set of functions in the inner region. We will verify that at the end of the calculations the regulators ϵ, ϵ' cancel independently in the results. We also define

$$
\nu' = l+1+\epsilon' \tag{3.19}
$$

To summarize, in the region $x \ll \frac{(Q_1+Q_5)R^2}{Q_1Q_5}$, the solution of the radial equation (3.11) is

$$
H_{in}(x) = x^\alpha(\gamma^2+x)^\beta F\left(p,q;1+2\alpha;-\frac{x}{\gamma^2}\right) \tag{3.20}
$$

We will need the asymptotic form of this expression for large values of x. To get this asymptotics it is convenient to rewrite (3.15) using an identity relating hypergeometric functions:

$$\begin{aligned} G(x) &= \frac{\Gamma(1+2\alpha)\Gamma(-\nu')}{\Gamma(\frac{1}{2}+\alpha+\beta-\frac{\nu'}{2})\Gamma(\frac{1}{2}+\alpha-\beta-\frac{\nu'}{2})}\left(\frac{x}{\gamma^2}\right)^{-p} F\left(p,p-2\alpha;\nu'+1;-\frac{\gamma^2}{x}\right) \\ &+ \frac{\Gamma(1+2\alpha)\Gamma(\nu')}{\Gamma(\frac{1}{2}+\alpha+\beta+\frac{\nu'}{2})\Gamma(\frac{1}{2}+\alpha-\beta+\frac{\nu'}{2})}\left(\frac{x}{\gamma^2}\right)^{-q} F\left(q,q-2\alpha;-\nu'+1;-\frac{\gamma^2}{x}\right) \end{aligned} \tag{3.21}$$

We will now use the fact that $p \approx q \approx \beta \gg 1$, since we take γ very small so that we have a very long throat. Let us look at the following series:

$$F(p,p-2\alpha;c;-z) = \sum_{n=0}^{\infty}\frac{\Gamma(p+n)}{\Gamma(p)}\frac{\Gamma(p-2\alpha+n)}{\Gamma(p-2\alpha)}\frac{\Gamma(c)}{\Gamma(c+n)}\frac{(-z)^n}{n!} \tag{3.22}$$

We are interested in a case where $p \to \infty$, while xp^2 remains bounded. Thus in the above expression we can replace

$$\frac{\Gamma(p+n)}{\Gamma(p)} \qquad \text{by} \qquad p^n \tag{3.23}$$

and we get

$$F(p,p-2\alpha;c;-z) \approx \sum_{n=0}^{\infty}\frac{\Gamma(c)}{\Gamma(c+n)}\frac{(-zp^2)^n}{n!} = (p\sqrt{z})^{-(c-1)}\Gamma(c)J_{c-1}(2p\sqrt{z}) \tag{3.24}$$

Then (3.21) can be approximated by

$$\begin{aligned} G(x) &\approx \frac{\Gamma(1+2\alpha)\Gamma(-\nu')\Gamma(\nu')\nu'}{\Gamma(\frac{1}{2}+\alpha+\beta+\frac{\nu'}{2})\Gamma(\frac{1}{2}+\alpha-\beta+\frac{\nu'}{2})}\left(\frac{x}{\gamma^2}\right)^{-\frac{n-m+1}{2}-\frac{\omega}{2\gamma}} p^{-\nu'} \\ &\times \left[\frac{\Gamma(\frac{1}{2}+\alpha+\beta+\frac{\nu'}{2})\Gamma(\frac{1}{2}+\alpha-\beta+\frac{\nu'}{2})}{\Gamma(\frac{1}{2}+\alpha+\beta-\frac{\nu'}{2})\Gamma(\frac{1}{2}+\alpha-\beta-\frac{\nu'}{2})}J_{\nu'}\left(\frac{2\gamma p}{\sqrt{x}}\right) - (pq)^{\nu'}J_{-\nu'}\left(\frac{2\gamma q}{\sqrt{x}}\right)\right] \end{aligned} \tag{3.25}$$

Using the relations $\gamma^2 p^2 \approx \gamma^2 q^2 \approx \omega^2/4$, we can rewrite the last expression as

$$G(x) \approx D_1 J_{\nu'}\left(\frac{\omega}{\sqrt{x}}\right) + D_2 J_{-\nu'}\left(\frac{\omega}{\sqrt{x}}\right), \tag{3.26}$$

and the radial wavefunction as

$$H_{in}(x) \approx \frac{1}{\sqrt{x}}\left[D_1 J_{\nu'}\left(\frac{\omega}{\sqrt{x}}\right) + D_2 J_{-\nu'}\left(\frac{\omega}{\sqrt{x}}\right)\right], \tag{3.27}$$

where

$$\frac{D_1}{D_2} = -(pq)^{-\nu'}\frac{\Gamma(\frac{1}{2}+\alpha+\beta+\frac{\nu'}{2})\Gamma(\frac{1}{2}+\alpha-\beta+\frac{\nu'}{2})}{\Gamma(\frac{1}{2}+\alpha+\beta-\frac{\nu'}{2})\Gamma(\frac{1}{2}+\alpha-\beta-\frac{\nu'}{2})} \tag{3.28}$$

Note that if we were solving the problem without rotation (i.e. $\gamma = 0$), then (3.27) would give the solution for the entire inner region (which in that case becomes an infinite throat), although we would not be able to find any relation between D_1 and D_2 since there would be no natural boundary condition from the end of the throat $x = 0$.

It is convenient to rewrite the RHS of (3.28) in terms of gamma functions with large positive arguments. Using the expression

$$\Gamma(x) = \frac{\pi}{\sin \pi x} \frac{1}{\Gamma(1-x)}, \tag{3.29}$$

we get:

$$\frac{D_1}{D_2} = -(pq)^{-\nu'} \frac{\Gamma(\frac{1}{2}+\alpha+\beta+\frac{\nu'}{2})\Gamma(\frac{1}{2}+\beta-\alpha+\frac{\nu'}{2})}{\Gamma(\frac{1}{2}+\alpha+\beta-\frac{\nu'}{2})\Gamma(\frac{1}{2}+\beta-\alpha-\frac{\nu'}{2})} \frac{\sin[\pi(\alpha-\beta+(1-\nu')/2)]}{\sin[\pi(\alpha-\beta+(1+\nu')/2)]} \tag{3.30}$$

To extract the leading order in $1/\beta$ we use an approximate relation

$$\frac{\Gamma(\frac{1}{2}+\alpha+\beta+\frac{\nu'}{2})}{\Gamma(\frac{1}{2}+\alpha+\beta-\frac{\nu'}{2})} \approx \beta^{\nu'}, \tag{3.31}$$

then using the expression (3.19) for ν', we can rewrite the ratio (3.30) as

$$\frac{D_1}{D_2} = (-1)^l \frac{\sin[\pi(\beta-\alpha+l/2+\epsilon'/2)]}{\sin[\pi(\beta-\alpha+l/2-\epsilon'/2)]} \tag{3.32}$$

which can be rewritten as

$$\frac{D_1 - D_2(-1)^l}{D_2(-1)^l} = \pi\epsilon' \frac{\cos[\pi(\beta-\alpha+l/2)]}{\sin[\pi(\beta-\alpha+l/2)]} \tag{3.33}$$

In the next subsection we will match the solutions between the inner and outer regions. To do this we will need the behavior of (3.27) for $x \gg \omega^2$ (which can be derived in the same way as the asymptotic expression (3.9))

$$H_{in}(x) \approx \frac{1}{\sqrt{x}} \left(\frac{\omega^2}{4x}\right)^{-\frac{l+1}{2}} \left[-(-1)^l D_2 \epsilon' l! + \frac{D_1 - (-1)^l D_2}{(l+1)!} \left(\frac{\omega^2}{4x}\right)^{l+1} \right] \tag{3.34}$$

3.4 Matching solutions in the intermediate region.

Let us compare the solutions (3.9) and (3.34). Requiring that

$$H_{out}(x) \approx H_{in}(x) \tag{3.35}$$

we get

$$\frac{C_2}{C_1 - (-1)^l C_2} = \frac{D_1 - D_2(-1)^l}{D_2} \frac{1}{\epsilon\epsilon'} \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2 \tag{3.36}$$

$$C_2 = \frac{D_2 - (-1)^l D_1}{\epsilon l!(l+1)!} \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{\frac{l+1}{2}} \tag{3.37}$$

3.5 The nature of the boundary condition at $r = 0$

One might at first think that since the geometry has a conical defect at $r = 0$ one will need to choose one out of several possible boundary conditions at this point before the wave problem is well defined. But we have seen that the boundary condition is automatically determined from the nature of the equation. At an algebraic level, this happened because the singularity is only at $r = 0, \theta = \pi/2$ rather than for all points with $r = 0$. This might still have implied that we need a boundary condition at $r = 0, \theta = \pi/2$, but the equation factorizes between r and θ, so that demanding regularity of the solution at $r = 0, \theta \neq \pi/2$ determines the behavior also at $r = 0, \theta = \pi/2$.

It would however be good to understand more physically how the wave turned back from $r = 0$. To do this let us analyze the wave equation (2.18) for $x \to 0$. We replace an expression like $x + \gamma^2$ by γ^2. Then the equation becomes

$$4\gamma^2 \frac{d}{dx}\left(x\frac{dH}{dx}\right) + \left\{(\omega^2 - \lambda^2)\frac{Q_1 + Q_5}{R^2} + \frac{(\omega - m\gamma)^2}{\gamma^2} - \Lambda - \frac{(\lambda + n\gamma)^2}{x}\right\} H = 0 \quad (3.38)$$

Writing $z = -\log x$, we can write the above equation as

$$-\frac{d^2H}{dz^2} + V(x)H = 0 \quad (3.39)$$

$$V(x) = \frac{(\lambda + n\gamma)^2}{4\gamma^2} - \frac{e^{-z}}{4\gamma^2}[(\omega^2 - \lambda^2)\frac{Q_1 + Q_5}{R^2} + \frac{(\omega - m\gamma)^2}{\gamma^2} - \Lambda] \equiv P - Qe^{-z} \quad (3.40)$$

Thus we have a Schroedinger equation with the above potential, with total energy $E = 0$. $x \to 0$ corresponds to $z \to \infty$. Note that $P \geq 0$. If $P > 0$, the wavefunction with $E = 0$ will automatically reflect back from the potential barrier back towards larger r. If $P = 0$, then we can solve the equation by writing $H = 1 + c_1 e^{-z} + c_2 e^{-2z} + \ldots$ and obtain a convergent expansion. The wavefunction is real, and no flux is carried out to $r = 0$ so that we see again that we get a reflection from $r = 0$.

This situation is to be contrasted with the equation we get when there is no angular momentum and the throat is infinitely long. Then for small r the equation takes the form

$$\frac{d}{dx}(x^2\frac{dH}{dx}) + \frac{\kappa}{x}H = 0 \quad (3.41)$$

with $\kappa = \frac{1}{4}[\omega^2 - \lambda^2]$. Writing $y = 1/x$ we get a Schroedinger equation of the form

$$-\frac{d^2H}{dy^2} - \frac{\kappa}{y}H = 0 \quad (3.42)$$

As we have seen, the solution can be expressed in terms of Bessel functions, and a wave incident from infinity becomes oscillatory at $y \to \infty$, carrying flux in towards $r = 0$.

Roughly speaking we may say that if a region of the geometry pinches off too sharply (as at the conical defect) then the wave reflects back from the walls of the throat near the pinch, while if it narrows more slowly then the wave continues along the throat, possibly gaining in amplitude, but in any case carrying flux in towards $r = 0$. Note however that if we depart from the supergravity approximation and require stringy effects to set in at string length, then it is possible to modify the above found behavior of the wavefunction in a region of radius string length around $r = 0$. In that case the incident wave could be absorbed at the conical defect and possibly return with some knowledge of the quantum state at the defect. (A similar reflection from the singularity for a different rotating system was studied in [11]).

4 Phase shift in the far outer region.

We have solved the low energy wave equation in the metric (2.1). From this solution we wish to extract the nature of scattering in this geometry; more precisely, we wish to use the method of phase shifts to compute a time delay and thus see how long the particle stays in the throat. Before we analyze the phase shifts it is helpful to understand the physics we expect, and to do this we first solve a toy model that produces similar phase shifts.

A quantum incident from infinity enters the throat with a small probability; the rest of the wave reflects back to infinity from the start of the throat. The wave which does enter the throat travels to its end, reflects back, but then has a small probability to emerge from the throat out to infinity. The rest of this trapped wave travels back down the throat again. The emerging wave thus has a large proportion with time delay zero – this is the part that never entered the throat. A small part will have time delay $2L$, where L is the length of the throat in the coordinate where the speed is unity. But there will be additional parts with time delays $4L, 6L, \ldots$. It is easy to see that the parts with time delay will have amplitude squared proportional $\mathcal{A}^2$, where $\mathcal{A}$ is the probability to enter or leave the throat.

We model this situation by a 1+1 dimensional Klein-Gordon equation in the following way. We put a delta function potential barrier at $x = 0$ which reflects most of the wave incident from $x > 0$, but allows a small part to enter the region $x < 0$. This latter region is the analogue of the throat. At $x = -L$ we put a reflecting wall, modeling the end of the throat.

4.1 Toy problem.

Let us consider a toy system which is governed by an analog of the two dimensional Klein–Gordon equation:

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\Phi(x,t) + V(x)\Phi(x,t) = 0 \tag{4.1}$$

with potential $V(x)$ given by

$$V(x) \quad = \quad c\delta(x), \qquad x > -L \tag{4.2}$$

$$V(x) \;=\; \infty, \qquad x < -L \tag{4.3}$$

We will look for the solution in the form: $\Phi(x,t) = e^{-ikt}\Psi(x)$. Then (4.1) can be rewritten as

$$-\Psi''(x) + V(x)\Psi(x) = k^2\Psi(x) \tag{4.4}$$

We can solve this equation in three different regions:

$$\begin{aligned}
x > 0: \qquad & \Psi(x) \;=\; Ae^{ikx} + Be^{-ikx}, && (4.5)\\
0 > x > -L: \qquad & \Psi(x) \;=\; Ce^{ikx} + De^{-ikx}, && (4.6)\\
x < -L: \qquad & \Psi(x) \;=\; 0. && (4.7)
\end{aligned}$$

and use boundary condition at $x = 0$ to write A and B in terms of C and D:

$$A = C - \frac{ic}{2k}(C+D), \qquad B = D + \frac{ic}{2k}(C+D). \tag{4.8}$$

In particular we will need the ratio

$$R \equiv \frac{A}{B} = \left(1 + \frac{2ik}{c}\frac{C}{C+D}\right)\left(-1 + \frac{2ik}{c}\frac{D}{C+D}\right)^{-1}. \tag{4.9}$$

The boundary condition $\Psi = 0$ at $x = -L$ gives:

$$C = -e^{2ikL}D \tag{4.10}$$

Then we get for the ratio (4.9):

$$R = \left(1 - \frac{2ik}{c}\frac{e^{2ikL}}{1-e^{2ikL}}\right)\left(-1 + \frac{2ik}{c}\frac{1}{1-e^{2ikL}}\right)^{-1} \tag{4.11}$$

Let us rewrite the ratio (4.11) in the following form:

$$\begin{aligned}
R &= -\left(\frac{1}{1-\frac{2ik}{c}} - e^{2ikL}\frac{1+\frac{2ik}{c}}{1-\frac{2ik}{c}}\right)\left(1 - \frac{e^{2ikL}}{1-\frac{2ik}{c}}\right)^{-1}\\
&= -\left(\frac{1}{1-\frac{2ik}{c}} - e^{2ikL}\frac{1+\frac{2ik}{c}}{1-\frac{2ik}{c}}\right)\sum_{n=0}^{\infty}\left(\frac{e^{2ikL}}{1-\frac{2ik}{c}}\right)^{n}\\
&= -\frac{1}{1-\frac{2ik}{c}} + \left(\frac{2k}{c}\right)^2\sum_{n=1}^{\infty}\frac{e^{2iknL}}{\left(1-\frac{2ik}{c}\right)^{n+1}}
\end{aligned} \tag{4.12}$$

This gives the expression for the wavefunction Φ_k corresponding to the given frequency k:

$$\Phi_k(x,t) = B\left[e^{-ikt-ikx} + e^{-ikt+ikx}\left(-\frac{1}{1-\frac{2ik}{c}} + \left(\frac{2k}{c}\right)^2\sum_{n=1}^{\infty}\frac{e^{2iknL}}{\left(1-\frac{2ik}{c}\right)^{n+1}}\right)\right] \tag{4.13}$$

If we look at a wave packet, for example,

$$\Phi(x,t) = \int dk \Phi_k(x,t) e^{-k^2/\lambda}, \tag{4.14}$$

we will see the incoming wave with a peak at $x = -t$ as well as various outgoing waves. The leading outgoing wave has a peak at $x = t$, and it corresponds to scattering back from the delta function potential. But there are also "secondary waves" with $x = t - 2Ln$, which correspond to scattering from the wall at $x = -L$. At the leading order in $\frac{2k}{c}$ the probability of going beyond $x = 0$ and coming back is given by

$$P_1 = \left(\frac{2k}{c}\right)^4. \tag{4.15}$$

We can also find the probability S_1 of an incident particle to proceed beyond $x = 0$ by comparing B with D. The calculation is done in the Appendix B and the result is $S_1 = \sqrt{P_1}$, as we could anticipate on physical grounds.

4.2 Phase shift for the D1–D5 system.

Let us go back to the solution in the outer region (3.2)

$$H_{out}(x) = \frac{1}{\sqrt{x}} \left(C_1 J_\nu \left(\sqrt{\frac{Q_1 Q_5 \omega^2 x}{R^4}} \right) + C_2 J_{-\nu} \left(\sqrt{\frac{Q_1 Q_5 \omega^2 x}{R^4}} \right) \right). \tag{4.16}$$

At very large x we can use the asymptotics of the Bessel's functions:

$$J_\nu(z) = \sqrt{\frac{2}{\pi z}} \cos(z - \frac{\pi\nu}{2} - \frac{\pi}{4}) + O(z^{-1}), \tag{4.17}$$

then we get:

$$\begin{aligned} H_{out} &= \frac{1}{x^{3/4}} \left(\frac{4R^4}{Q_1 Q_5 \omega^2 \pi^2} \right)^{1/4} \left[C_1 \cos\left(\omega y - \frac{\pi\nu}{2} - \frac{\pi}{4} \right) + C_2 \cos\left(\omega y + \frac{\pi\nu}{2} - \frac{\pi}{4} \right) \right] \\ &+ O(x^{-3/4-1/2}) \end{aligned} \tag{4.18}$$

Here we have introduced a natural variable in the far outer region:

$$y = \sqrt{\frac{Q_1 Q_5 x}{R^4}} \tag{4.19}$$

We now need to compare the phases of the ingoing and outgoing waves. Ignoring the overall y dependence of the wavefunction we have

$$\begin{aligned} H_{out} &\sim e^{i\omega y - i\pi\frac{l}{2} - \frac{3\pi i}{4}} \left(C_1 e^{-i\pi\epsilon/2} - C_2 (-1)^l e^{i\pi\epsilon/2} \right) \\ &+ e^{-i\omega y + i\pi\frac{l}{2} + \frac{3\pi i}{4}} \left(C_1 e^{i\pi\epsilon/2} - C_2 (-1)^l e^{-i\pi\epsilon/2} \right) \end{aligned} \tag{4.20}$$

Since we are looking for a long time delay we can ignore the finite shifts in y included in the exponentials in the above expression. Following what we did in the toy problem we consider the ratio:

$$R = \frac{C_1 e^{-i\pi\epsilon/2} - C_2(-1)^l e^{i\pi\epsilon/2}}{C_1 e^{i\pi\epsilon/2} - C_2(-1)^l e^{-i\pi\epsilon/2}} \tag{4.21}$$

Note that for real C_1 and C_2 we get $|R| = 1$, in accordance with the fact that there is no loss of flux down the throat. We rewrite R in the following form:

$$\begin{aligned} R &= e^{-i\pi\epsilon} - (1 - e^{-2i\pi\epsilon}) \frac{C_2(-1)^l}{C_1 - C_2(-1)^l e^{-i\pi\epsilon}} \\ &\approx e^{-i\pi\epsilon} - 2i\pi\epsilon \frac{D_1 - D_2(-1)^l}{D_2(-1)^l} \frac{1}{\epsilon\epsilon'} \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2 \\ &\approx e^{-i\pi\epsilon} - 2i\pi^2 \frac{\cos\left[\pi\left(\beta - \alpha + l/2\right)\right]}{\sin\left[\pi\left(\beta - \alpha + l/2\right)\right]} \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2 \end{aligned} \tag{4.22}$$

where in the second step we have used the expression (3.36) to relate C and D, and in the last step we have used (3.33).

To find the different phase shifts contained in the scattered wave we write the above expression in a form similar to (4.13)

$$R \approx e^{-i\pi\epsilon} - 2\pi^2 \frac{1 + e^{2\pi i(\beta - \alpha + l/2)}}{1 - e^{2\pi i(\beta - \alpha + l/2)}} \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2 \tag{4.23}$$

and make a formal expansion of the term $1 - e^{2\pi i(\beta - \alpha + l/2)}$ in the denominator:

$$\begin{aligned} R &\approx \left[e^{-i\pi\epsilon} - 2\pi^2 \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2\right] \\ &- 4\pi^2 \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2 \sum_{n=1}^{\infty} e^{2\pi i n(\beta - \alpha + l/2)} \end{aligned} \tag{4.24}$$

From the phases of the terms in the infinite sum we read off the time delay between the emerging wave–packets:

$$\Delta t = 2\pi \frac{\partial}{\partial \tilde{\omega}}(\beta - \alpha) = \pi \frac{R}{\gamma} = \pi \frac{\sqrt{Q_1 Q_5}}{a}. \tag{4.25}$$

The coefficient of the scattered waves yields the probability of going into the throat and coming back

$$P_2 = \left[4\pi^2 \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2\right]^2 \tag{4.26}$$

We can also evaluate the probability S_2 for going inside the throat. As in the case of the toy model we get $P_2 = S_2^2$. The details of the calculation are presented in Appendix B.

The result should agree with the known probability of absorption into the near extremal D1-D5 geometry [4], where the particle enters the throat and then proceeds inwards without returning. As a cross check on our calculations we verify that we indeed obtain the correct probability.

5 Properties of the 'hot tube'.

5.1 Some scales in the D1-D5 system

Let us first recall the relations between the microscopic D1-D5 system (with no angular momentum) and the geometry that it produces. We set the momentum charge n_p of the system to zero, but allow a small nonextremality characterized by the parameter r_0. We follow for the most part the notations of [12], [4].

The 10-d Einstein metric and dilaton are

$$\begin{aligned} ds^2 &= (1+\frac{Q_1}{r^2})^{-3/4}(1+\frac{Q_5}{r^2})^{-1/4}\left[-(1-\frac{r_0^2}{r^2})dt^2+dx_5^2+(1+\frac{Q_1}{r^2})dx_idx_i\right] \\ &+ (1+\frac{Q_1}{r^2})^{1/4}(1+\frac{Q_5}{r^2})^{3/4}\left[(1-\frac{r_0^2}{r^2})dr^2+r^2d\Omega_3^2\right] \end{aligned} \quad (5.1)$$

$$e^{-2\phi}=(1+\frac{Q_1}{r^2})^{-1}(1+\frac{Q_5}{r^2}) \quad (5.2)$$

We write

$$\begin{aligned} Q_1 &\equiv r_1^2 = r_0^2\sinh^2(2\alpha_1) \\ Q_5 &\equiv r_5^2 = r_0^2\sinh^2(2\alpha_5) \end{aligned} \quad (5.3)$$

We work in units where $\alpha'=1$. The volume of the T^4 spanned by the coordinates x_i is $(2\pi)^4V$, and the length of the circle x_5 is $2\pi R$. The 10-d Newton's constant is $G_{10}=8\pi^6g^2$. The extremal configuration is given by $r_0\to 0, \alpha_1\to\infty, \alpha_5\to\infty$, while holding Q_1, Q_5 fixed. We will work in the near extremal limit $r_0\ll r_1, r_5$.

The number of D1 branes and D5 branes that produce the above solution are respectively

$$n_1=\frac{Vr_0^2}{2g}\sinh(2\alpha_1)\approx\frac{VQ_1}{g} \quad (5.4)$$

$$n_5=\frac{r_0^2}{2g}\sinh(2\alpha_5)\approx\frac{Q_5}{g} \quad (5.5)$$

The mass of the solution is

$$M=\frac{RVr_0^2}{2g^2}(\cosh(2\alpha_1)+\cosh(2\alpha_5)+1) \quad (5.6)$$

where the last term in the bracket arises from a term $\cosh(2\alpha_p)$ when we set $\alpha_p = 0$. (The parameter α_p gives the momentum charge through $n_p = (R^2 V r_0^2/2g^2)\sinh(2\alpha_p)$.) For small nonextemality we find that the mass above extremality is essentially given by the last term in the above relation [13]

$$\delta M \equiv M - M_{extremal} \approx \frac{RVr_0^2}{2g^2} \tag{5.7}$$

For our case of no momentum charge the left and right temperatures T_L, T_R of the system equal the Hawking temperature T_H

$$T_L = T_R = T_H = \frac{r_0}{2\pi r_1 r_5}. \tag{5.8}$$

5.2 The length scale Rn_1n_5

The thermodynamic properties of the D1-D5 system are well described by an 'effective string' which is 'multiply wound' around $x_5 = y$. We are interested in the ground state which corresponds to one single long string; this string has a total effective length $L_{eff} = n_1 n_5 L$, where $L = 2\pi R$ is the length of the x_5 circle. It was argued in [14] that the low energy excitations of a multiwound string are harmonic vibrations that carry left and right momentum in units of $2\pi/L_{eff}$, but that the net momentum of the system must still be an integer multiple of $2\pi/L$. The lowest excitation thus has one left and one right moving mode, with net momentum zero and a total energy

$$\delta M = \frac{4\pi}{L_{eff}} = \frac{2}{n_1 n_5 R} \tag{5.9}$$

It was argued in [15] that the above energy scale is a physically relevant scale emerging from the thermodynamics of the D1-D5 geometry: if the nonextremal energy is not much larger than (5.9) then the temperature suffers a fractional change of order unity when a single typical quantum is emitted, and the emission process ceases to be described by naive thermodynamics [17]. We may verify this fact as follows. We start from the extremal system ($r_0 = 0$), throw in a quantum of energy $\tilde{\omega}$ and ask that the temperature increase from zero to order $\tilde{\omega}$. We have

$$\tilde{\omega} = \delta M \sim \frac{RVr_0^2}{g^2} \tag{5.10}$$

while

$$T_H \sim \frac{r_0}{r_1 r_5}. \tag{5.11}$$

Requiring $\tilde{\omega} \sim T_H$ gives the desired result

$$\tilde{\omega} \sim \frac{1}{Rn_1n_5}. \tag{5.12}$$

For future use, let us define

$$\tilde{\omega}_0 \equiv \frac{1}{Rn_1n_5}. \tag{5.13}$$

5.2.1 The length $n_1 n_5 R$ from the length of the throat

The unexcited D1-D5 system can be in one of several Ramond ground states, all of which have the same energy. These Ramond (R) ground states can be obtained by spectral flow of chiral primary states from the Neveu-Schwarz (NS) sector. The chiral primary states can be placed in a finite number of classes, based on which cohomology element we pick from the compact 4-manifold (which can be T^4 or $K3$). Let us consider the description of states at the orbifold point of the D1-D5 system. The states involve twist operators σ_n of order n. A basic family of states, studied in [18], has dimension and SU(2) charge $h = j_3 = \frac{n-1}{2} = j$. After we spectral flow such a state to the R sector we get $h = \frac{c}{24}, j_3 = \frac{n-1}{2} - \frac{c}{12}$. Since $c = 6N, N = n_1 n_5$, we get $h = \frac{N}{4}$ and

$$j_3 = \frac{n-1-N}{2} \tag{5.14}$$

The R sector states arrange themselves into multiplets of SU(2) with (5.14) as the lowest value of the j_3 charge. But since $1 \leq n \leq N$, we see that at least for this family of states the smallest value of SU(2) charge is $j = 1/2$ and not $j = 0$.

Note that this SU(2) charge is just the angular momentum of the state. But if the angular momentum is given by $\gamma \neq 0$ then we know that the throat of the corresponding geometry is not infinite but in fact truncated at some distance with the formation of a conical defect. Let us set the angular momentum to be $j = 1/2$, so that

$$\gamma = \frac{j}{j_{max}} = \frac{\frac{1}{2}}{\frac{1}{2} n_1 n_5} = \frac{1}{n_1 n_5} \equiv \gamma_0 \tag{5.15}$$

Let us now ask how long the throat is for this value of γ. We imagine a massless particle thrown into the throat, and ask how long it takes to reach the end. Note that $\gamma = \gamma_0$ gives a very small angular momentum to the system, so we may use the geometry without angular momentum to compute the time of flight of the particle, if we set $x \approx \gamma_0^2$ as the end of the throat. This corresponds to $r \approx \gamma_0 \sqrt{Q_1 Q_5}/R$. We set $Q_1 \sim Q_5$. The start of the throat may be taken as $r \sim (Q_1 Q_5)^{1/4}$. The time of flight is then given by setting $ds^2 = 0$, which gives

$$t = \int_{r_{min}}^{r_{max}} dr \frac{g_{rr}^{1/2}}{-g_{tt}^{1/2}} \approx \int_{\gamma_0 \sqrt{Q_1 Q_5}/R}^{(Q_1 Q_5)^{1/4}} \sqrt{Q_1 Q_5}\, \frac{dr}{r^2} \approx n_1 n_5 R \tag{5.16}$$

Thus we see that the length of the tube in the geometry with minimum angular momentum gives the wavelength scale associated to the point where thermodynamic behavior of the near extremal D1-D5 system breaks down.

5.2.2 Horizon formation

Consider the geometry of the extremal D1-D5 system with no rotation ($\gamma = 0$). Let us imagine that we throw in a massless quantum with energy ω, with a spin $j = 1/2$. As the quantum travels down the throat it can reach a point where its mass will cause a horizon to form. But because the quantum carries nonzero j, the geometry outside the location of the quantum will exhibit this value of angular momentum and so the throat can at some point end in a conical defect. Let us ask if the horizon will form first or the conical defect will form first. If the conical defect forms before the horizon, then we may speculate that the particle will bounce back along the throat rather than disappear behind a horizon, though we cannot prove this rigorously since we are not explicitly solving for the backreaction created by the quantum.

To locate the point of horizon formation we need to find the value of r_0 when the nonextremal mass given to the D1-D5 system is $\tilde{\omega}$. Setting

$$\tilde{\omega} = \delta M \sim \frac{RVr_0^2}{g^2} \tag{5.17}$$

we get

$$r_0^2 \sim \frac{g^2\tilde{\omega}}{RV} \tag{5.18}$$

which corresponds to

$$x = \frac{R^2}{Q_1Q_5}r_0^2 \sim \frac{1}{(n_1n_5)^2} = \gamma_0^2\frac{\tilde{\omega}}{\tilde{\omega}_0} \tag{5.19}$$

Thus we observe that if $\tilde{\omega} = \tilde{\omega}_0 = 1/Rn_1n_5$ then the horizon forms at $x \sim \gamma_0^2$, which is the same point where the conical defect would terminate the tube because of the angular momentum carried by the quantum. If $\tilde{\omega} > \tilde{\omega}_0$ (and the particle still has $j = 1/2$) then the horizon forms before the rotation terminates the tube. For energies $\tilde{\omega} < \tilde{\omega}_0$ we may speculate that the quantum returns back without horizon formation, which would agree with the fact that in the microscopic picture of the 'effective string' energies lower than $1/Rn_1n_5$ cannot be absorbed.

5.2.3 Nonlinearity of the wave equation

Let us now arrive at the length scale Rn_1n_5 in yet another way. Consider the geometry of the unexcited D1-D5 system with $\gamma = 0$; the geometry has an infinite throat. Let us throw in one quantum of a scalar field.

The infall of the quantum down the throat can be described, at least for some time, by the linear wave equation studied in the last section. But as the wave moves down the throat and acquires an oscillatory character, we see that its amplitude also increases. This can be seen from the behavior of the solution in terms of Bessel's functions valid for the nonrotating geometry in the throat region. The solution behaves as

$$\phi \sim D_1\frac{1}{\sqrt{x}}J_\nu(\frac{\omega}{\sqrt{x}}) + D_2\frac{1}{\sqrt{x}}J_{-\nu}(\frac{\omega}{\sqrt{x}}) \tag{5.20}$$

But for large argument the Bessel's functions itself behaves as

$$J_\nu(z) \approx \sqrt{\frac{2}{\pi z}}\cos(z - \frac{\pi}{2}\nu - \frac{\pi}{4}) \tag{5.21}$$

so that we see that the amplitude of the wave grows in the throat as

$$x^{-1/4} \sim r^{-1/2} \tag{5.22}$$

The start of the throat is at $r \sim (Q_1Q_5)^{1/4}$. Let us follow the quantum down the throat for the time Rn_1n_5, which brings us to $r \sim \gamma_0\sqrt{Q_1Q_5}/R$. This implies an amplification factor

$$\mu = [\frac{\gamma_0(Q_1Q_5)^{1/4}}{R}]^{-1/2} \tag{5.23}$$

To see whether the wave has at this point become essentially nonlinear, we must know the initial strength of the wave. To estimate this we write the action for the scalar as

$$S = \frac{1}{16\pi G_N}\int d^{10}x\frac{1}{2}\partial\phi\partial\phi \tag{5.24}$$

and we obtain the Hamiltonian from this action. The energy of a quantum is $\tilde{\omega}$. To estimate the value of ϕ for such a quantum at the start of the throat, we need to know the volume d^9x occupied by the wavepacket. We may confine the wavepacket to a longitudinal distance of order $\tilde{\omega}^{-1}$, while the transverse area is made up of V (the volume of the T^4), R (the length of y), and the area of S^3 which at the start of the throat is $\sim (Q_1Q_5)^{3/4}$. Thus we get

$$\frac{1}{G_N}\tilde{\omega}^{-1}VR(Q_1Q_5)^{3/4}(\tilde{\omega}^2\phi^2) \sim \tilde{\omega} \tag{5.25}$$

Note that $\tilde{\omega}$ cancels out, and we get (using $G_N \sim g^2$)

$$\phi \sim [\frac{g^2}{RV(Q_1Q_5)^{3/4}}]^{1/2} \tag{5.26}$$

Multiplying by the amplification factor (5.23) we find that the value of ϕ at the point $x \sim \gamma_0^2$ is

$$\phi \sim 1 \tag{5.27}$$

so that the linear wave equation can become invalid after the quantum travels a distance Rn_1n_5 down the throat.

6 Making the D1-D5 system into a 'hot tube'

We have seen that a quantum traveling down the throat of the D1-D5 system reflects off the end if $\gamma \neq 0$. The quantum then travels back towards the start of the throat. Our calculations were done for low energies, so that the wavelength of the incoming quantum was much larger than the length scale $(Q_1Q_5)^{1/4}$ describing the scale of variation of the geometry at the start of the throat. When such a low energy quantum is thrown in from infinity, then there is a small probability that it enters the throat. Let us restrict our attention to s-waves ($l = 0$). The absorption probability is $\mathcal{A} \sim \tilde{\omega}^4Q_1Q_5 \ll 1$. But this also implies that when the quantum reflects from the end of the throat and travels back to larger r then it has the same probability $\mathcal{A}$ to escape out of the throat to infinity. Thus the wave reflects back (with a probability close to unity) to small r again, and we see that the quantum is trapped in the throat for a long time. We can fill up the throat with several quanta, and in particular imagine that the configuration is placed in a thermal state. (Whether an actual initial configuration of quanta reaches thermal equilibrium before effusing out depends on the strength of interactions and the length of the tube.) We call such a thermal configuration a 'hot tube'. In this section we study some of the properties of these hot tubes. (In [16] the near extremal 5-brane was studied. The throat in this case was also filled with radiation, but it ended in a horizon since there was no rotation.)

6.1 Some preliminaries

Let us imagine that we fill up the tube with radiation that has a temperature T. The energy of a typical quantum will be $\tilde{\omega} \sim T$. We measure all energies in terms of the time coordinate at infinity, and recall that the frequency in this coordinate was called $\tilde{\omega} \equiv \omega/R$.

The length of the tube is determined by the value of the parameter γ; the smaller the value of γ, the longer the tube. We do not want to take too long a tube, since otherwise a horizon will form at some point in the tube, and we will have a black hole which swallows quanta rather than an endpoint that reflects them. Let the total nonextremal energy be E_T. Then the position of the horizon would be

$$r_0^2 \sim \frac{g^2 E_T}{RV}, \quad \rightarrow \quad x \sim \frac{g^2 R E_T}{V Q_1 Q_5} \tag{6.1}$$

(Of course if the radiation is distributed through the tube rather than concentrated at the end then not all the energy E_T will contribute to horizon formation. But it turns out that reducing the value of E_T to take into account this effect does not change the above estimates.) The tube ends at $x \sim \gamma^2$, so that we need

$$\gamma^2 > \frac{g^2 R E_T}{V Q_1 Q_5}. \tag{6.2}$$

Note that putting $E_T = (R n_1 n_5)^{-1}$ gives $\gamma \sim 1/n_1 n_5$.

We also do not want the tube to be too short since we want the wavelength of the typical quantum to fit into the tube. The solution of the wave equation in the throat was given in terms of Bessel's functions with argument $\frac{\omega}{\sqrt{x}}$, and these functions become oscillatory in the throat only for $\frac{\omega}{\sqrt{x}} > 1$. Since the throat ends at $x \sim \gamma^2$, we get the requirement

$$\gamma < \omega = R\tilde{\omega} \sim RT \tag{6.3}$$

6.2 The relation between E_T and T

We still need to find the relation between the energy added to the tube E_T and the temperature T which the tube attains as a consequence; i.e., we need to know the heat capacity of the tube. To do this it is helpful to recall the expression for the time of flight for a massless particle in the throat:

$$t = \sqrt{Q_1 Q_5} \int \frac{dr}{r^2} \tag{6.4}$$

Let us write

$$y = \frac{\sqrt{Q_1 Q_5}}{r} \tag{6.5}$$

Then the classical trajectory for a massless particle satisfies $dt = |dy|$ and the waveform behaves as $e^{-i\tilde{\omega}(t \pm y)}$. We have thus a regular one dimensional thermal system. The minimum value of y is $\sim (Q_1 Q_5)^{1/4}$ at the start of the throat, but we can set this to zero for our estimates. The end of the tube is at $y \sim \frac{R}{\gamma}$. The number of wavelengths of the typical quantum that fit in this interval of y is

$$n \sim \frac{R/\gamma}{\tilde{\omega}^{-1}} = \frac{R\tilde{\omega}}{\gamma} \tag{6.6}$$

The total energy of the gas of quanta is then

$$E_T \sim \tilde{\omega} n \sim \frac{R\tilde{\omega}^2}{\gamma} \sim \frac{R}{\gamma} T^2 \tag{6.7}$$

[If we consider instead the system with no rotation but a horizon at some $r = r_0$ then the nonextremal energy is related to the temperature by

$$E_{BH} \sim \frac{R}{\gamma_0} T^2, \quad \gamma_0 = \frac{1}{n_1 n_5} \tag{6.8}$$

It is curious that if we naively set $\gamma = \gamma_0$ (its minimum value) then we reproduce with the hot tube the order of the heat capacity of the black hole. But if we choose γ to be so small then the backreaction of the thermal quanta will lead to horizon formation in the tube and invalidate the tube analysis. It is possible that there is some deeper reason which tells us that we can ignore the backreaction and recover the properties of the black hole.]

6.3 Radiation from the hot tube

Quanta can slowly leak out of the throat of this hot tube just as radiation escapes to infinity from the near extremal black hole. The essential difference is of course that the quanta in the throat are generated by the horizon in the black hole case, while they have been added by hand for the hot tube. Thus the temperature of the hot tube is not determined by its geometry, while it is determined by geometry for a black hole.

We are looking at the $l = 0$ harmonics (where most of the radiation occurs). The probability for a quantum incident from infinity to enter the throat is $\mathcal{A}$. After the quantum enters the throat, it travels to a very good approximation as a free particle with the waveform $e^{-i\tilde{\omega}(t-y)}$, before reflecting from the end of the tube. Thus the probability for it to exit the tube is determined by local physics near the start of the throat ($r \sim (Q_1Q_5)^{1/4}$) and unrelated to the geometry at the end of the tube. We have therefore a free gas of massless quanta trapped in the tube, which gives a 1-dimensional thermal system. each time a quantum reaches back to the start of the throat it has a probability $\mathcal{A}$ of escaping to infinity. We now estimate the radiation rate from the tube.

Let each typical quantum have a wavelength λ. Let the effective length along the tube (measured in the coordinate y where $dt = |dy|$) be $L = n\lambda$. A given quantum reaches the start of the tube $\sim 1/n\lambda$ times per unit of the time t. There are $\sim n$ quanta of this typical wavelength in the thermal distribution so $1/\lambda \sim \tilde{\omega}$ quanta try to escape the tube per unit time. Since the escape probability is $\mathcal{A}$, the number of quanta exiting per unit time is

$$\Gamma \sim \tilde{\omega} A \tag{6.9}$$

Let us compare this rate to the emission rate from a near extremal black hole which has a temperature $T \sim \tilde{\omega}$. The number of quanta emitted per unit time is

$$\Gamma_{BH} = \frac{\sigma}{e^{\tilde{\omega}/T} - 1} \frac{d^4k}{(2\pi)^4} \tag{6.10}$$

But the absorption cross section σ is given by $\sigma = \frac{4\pi}{\tilde{\omega}^3}\mathcal{A}$ where $\mathcal{A}$ is the same probability as the one appearing in the calculation with the hot tube. Setting the Boltzmann factor to be order unity for a typical quantum (we did the same for the hot tube) we get

$$\Gamma_{BH} \sim \frac{\mathcal{A}}{\tilde{\omega}^3}\tilde{\omega}^4 \sim \tilde{\omega}\mathcal{A} \tag{6.11}$$

which is of the same order as emission from the hot tube. This is not surprising, since the emission from the throat depends on the temperature in the throat, and we have compared systems where this temperature is the same. We have seen above though that the heat capacity is in general different in the two cases.

7 Throat geometry and states of the D1-D5 CFT

We have seen above that the geometry of the throat reflects the properties of the D1-D5 bound state in the following way. The D1-D5 system can be modeled by an effective string that is wrapped n_1n_5 times around the circle of radius R. If all the strands of this string are joined together to make one long strand, then the lowest energy mode is $\tilde{\omega} \sim 1/Rn_1n_5$. We have seen that if we go a distance Rn_1n_5 down the throat of the corresponding geometry then special effects occur: a spin $j = 1/2$ would end the throat in a conical defect, and any scalar wave would tend to become nonlinear. But the D1-D5 system can be in a host of different states – the spin may not be $j = 1/2$ or all the strands of the effective string may not be joined into a single string. In this section we briefly discuss how the throat geometry might reflect these possibilities.

Spins along T^4:

Let the strands of the effective string be joined up into one single long string. This string has a multiplet of ground states. A subset of these ground states have spin j along the S^3 directions of order unity, and we have seen that this spin causes the throat to end at a distance $\sim n_1 n_5 R$. There are other members of the multiplet which have no spin along S^3, but have a spin along the directions of the compact space T^4. Let us consider such states.

We note that the S^3 gives the R symmetry group $SO(4)$, but in the T^4 the local symmetry is another $SO(4)$, which we call $SO(4)_I$. The compactification breaks rotational symmetry in the T^4, but for our rough analysis we ignore this fact. These two $SO(4)$'s are known to play very similar roles in the system, and so we speculate that a spin along the T^4 would cause the throat to truncate in the same way (and at the same approximate distance) as the spin $j = 1/2$ along the S^3.

Branching throats:

Now consider a state of the CFT where the $n_1 n_5$ strands of the effective string are not all joined together, but are in fact joined to make m different strings, each of total length $\frac{n_1 n_5}{m} R$. In this case the lowest energy absorbed by the state would be $m\tilde{\omega}_0 = \frac{m}{n_1 n_5 R}$. We now ask if the throat geometry could be such so as to return back (without horizon formation) all modes with energies lower than $m\tilde{\omega}_0$.

First consider the case where the spins of each string are along S^3, and further that these spins are all aligned. Then the state has angular momentum $j = m/2$, which gives $\gamma = m\gamma_0$. Recalling the calculation (5.16), and noting that r at the endpoint is linear in the value of γ, we see that in this case the time of travel to the end of the throat will be $\sim \frac{n_1 n_5 R}{m}$, and with the endpoint of the throat at this location, quanta with energy less than $m\tilde{\omega}_0$ will indeed return without horizon formation.

Now consider the case where the spins are not aligned, and in fact add together to give a total angular momentum $j \sim 0$. The CFT state still suggests that the minimum frequency that can be absorbed in $m\tilde{\omega}_0$ (and not $\tilde{\omega}_0$). But if we put $\gamma \sim 0$ or $\gamma \sim 1/2$ in the geometry then we find that the throat has an effective length $\sim n_1 n_5 R = \tilde{\omega}_0^{-1}$ or larger. In view of the fact that the CFT state was described by m strings it appears natural to consider that the throat itself branches at some distance into m throats, each with an effective value of $n_1 n_5$ given by $\frac{n_1 n_5}{m}$. Each throat also carries $j = 1/2$, and the spins of different branches are not necessarily aligned. This spin $j = 1/2$ causes each branch to terminate after a distance $\frac{n_1 n_5 R}{m}$, and we recover the desired low energy behavior.

Branching throat geometries were also considered in [19] with somewhat different motivations. It is possible that quanta with different spins and energies stay trapped at different points along the throat due to the backreaction that they produce, so we should not put their energy and charge together to create the standard black hole geometry. It may also be illuminating to study absorption by the dual geometry made from winding and momentum charges. Here we may see the string separating into several strings [20].

8 Discussion

The near horizon geometry of the D1-D5 system is locally $AdS_3 \times S^3$ [21]. If the direction y is taken noncompact, then the space $r > 0$ occupies a Poincare patch of the AdS spacetime. The geometry smoothly continues to $r < 0$ however, suggesting that there is at least one more region of spatial infinity[22]. This appears paradoxical, since we can assemble D1 and D5 branes in flat space to make the D1-D5 geometry, and it is hard to see how a new infinite region can emerge just by bringing together solitons in a space with only one spatial infinity.

When the direction y is compactified, the geometry is expected to become singular at $r = 0$ since the points identified under $y \to y + 2\pi R$ become closer and closer to each other as $r \to 0$. Thus we may speculate that in this quanta thrown towards $r = 0$ return back to the same spatial infinity where they came from. If a horizon forms, then we have the usual difficulties of tracking how the information escapes back to infinity. But modes that are sufficiently low in energy may return back without horizon formation.

The question therefore arises: what is the length scale which governs when such modes turn back? One may naively speculate that waves reflect when the distance between identified points $y, y + 2\pi R$ becomes smaller than string length or Planck length. But as we can see from our discussion this would be too short a return time scale to agree with the physics of the D1-D5 microstate. Our results suggest that the throat effectively terminates for different reasons, which are related to angular momentum and nonlinearity of the wave equation. Our results also suggest that the length of the throat would equal the length of the effective string in the D1-D5 CFT, and that a general description of the throat geometry might have to include branching of throats.

Finally, we turn to the question of what these results might say for the problem of information recovery in the case that a horizon does form. If throats can branch then they can also rejoin, and we imagine a complex web of quantum fluctuating virtual tubes that would replace the simple picture of a single infinite throat. In particular if a tube rejoins the geometry near but just outside the horizon, then it can bring information out to a point where it can escape to infinity. There is a cost in action to create a branching or a joining of tubes, but near the horizon this cost is low since the proper time for which a virtual tube lasts is small due to the redshift factor. We must of course face the usual difficulty that one can Lorentz transform to regular variables near the horizon and so nothing special can happen there, but such a boost does not make sense if the geometry has a complex topology of tubes interconnecting disparate locations. We hope to return to a study of this issue later.

A Corrections to the eigenvalues of the angular Laplacian.

In this paper we have used an approximate expression for the eigenvalue Λ:

$$\Lambda = l(l+2) \tag{A.1}$$

where l is non–negative integer. This expression becomes exact only for non–rotating system (i.e. for the metric (2.1) with $a = 0$). In this appendix we will look at the general equation (2.15) for eigenvalue Λ and we will find the leading order correction to Λ. For simplicity we will consider only solutions with $m = n = 0$. Then (2.15) becomes:

$$\frac{1}{\sin 2\theta}\frac{d}{d\theta}\left(\sin 2\theta \frac{d\Theta}{d\theta}\right) + a^2(\tilde{\omega}^2 - \tilde{\lambda}^2)\cos^2\theta\Theta = -\Lambda\Theta \tag{A.2}$$

It is convenient to introduce a new variable $z = \cos 2\theta$. Then the domain $0 \leq \theta \leq \pi/2$ becomes $-1 \leq z \leq 1$, and equation (A.2) becomes:

$$4\frac{d}{dz}\left((1-z^2)\frac{d\Theta}{dz}\right) + a^2(\tilde{\omega}^2 - \tilde{\lambda}^2)\frac{1+z}{2}\Theta = -\Lambda\Theta \tag{A.3}$$

We are looking for a solution of this equation which is normalizable on the interval $-1 \leq z \leq 1$.

We will treat a–dependent term in (A.3) as a small perturbation. First we note that (A.3) can be rewritten as a familiar quantum mechanical problem

$$(\hat{H}_0 + \hat{H}_1)|\Theta\rangle = E|\Theta\rangle \tag{A.4}$$

where

$$\hat{H}_0 \equiv 4\frac{d}{dz}\left((1-z^2)\frac{d}{dz}\right) \tag{A.5}$$

is the unperturbed Hamiltonian,

$$\hat{H}_1 \equiv a^2(\tilde{\omega}^2 - \tilde{\lambda}^2)\frac{1+z}{2} \tag{A.6}$$

is a perturbation, and

$$E \equiv -\Lambda. \tag{A.7}$$

Then the unperturbed equation $\hat{H}_0|\Theta\rangle = E_0|\Theta\rangle$ reads

$$\frac{d}{dz}\left((1-z^2)\frac{d\Theta}{dz}\right) + \frac{\Lambda_0}{4}\Theta = 0. \tag{A.8}$$

This is the equation for the Legendre polynomials, and the solution normalizable on the interval $(-1, 1)$ exists only for

$$\frac{\Lambda_0}{4} = s(s+1), \tag{A.9}$$

where s is a non–negative integer. Note that these eigenvalues correspond to even angular momentum in (2.16) ($l = 2s$), and to find the remaining eigenvalues one should consider the case where the difference $m - n$ is an odd integer. We will not consider this case here.

The eigenvalue $\Lambda_0 = 4s(s+1)$ corresponds to the eigenfunction

$$\Theta_s(z) = P_s(z), \tag{A.10}$$

where $P_s(z)$ is the Legendre polynomial. These polynomials form a complete system in the space of functions normalizable on the interval $(-1, 1)$.

To determine the leading order correction to Λ_0 we will go back to the quantum mechanical analogy (A.4) and recall that the lowest order correction to E is given by

$$\delta E = \frac{\langle\Theta_0|\hat{H}_1|\Theta_0\rangle}{\langle\Theta_0|\Theta_0\rangle}, \tag{A.11}$$

where $|\Theta_0\rangle$ is an eigenstate of the unperturbed Hamiltonian $\hat{H}_0$. Thus for our problem we get:

$$\delta\Lambda = -\left(\int_{-1}^{1} dz P_s(z) P_s^*(z)\right)^{-1} \int_{-1}^{1} dz a^2(\tilde{\omega}^2 - \tilde{\lambda}^2)\frac{1+z}{2} P_s(z) P_s^*(z) \tag{A.12}$$

Let us now use the symmetry properties of the Legendre polynomials:

$$\begin{aligned} P_s(-z) &= P_s(z) \qquad \text{for even } s, \\ P_s(-z) &= -P_s(z) \qquad \text{for odd } s. \end{aligned} \tag{A.13}$$

In particular, the product $P_s(z)P_s^*(z)$ is an even function of z for all s. Then using the facts that $f(z) = z$ is an odd function of z and that the ranges of integration in (A.12) are symmetric under $z \to -z$, we get a simple expression for $\delta\Lambda$:

$$\delta\Lambda = -\left(\int_{-1}^{1} dz P_s(z) P_s^*(z)\right)^{-1} \int_{-1}^{1} dz a^2(\tilde{\omega}^2 - \tilde{\lambda}^2)\frac{1}{2} P_s(z) P_s^*(z) = -\frac{a^2}{2}(\tilde{\omega}^2 - \tilde{\lambda}^2). \tag{A.14}$$

B Evaluation of the absorption probability.

In section 4 we have evaluated probabilities of reaching and bouncing off the infinite wall (for the toy system) and off the conical defect at the end of the throat. We called these probabilities P_1 and P_2. Here we will evaluate the probability S_1 for a quantum incident from infinity to pass through the initial barrier and enter the region $x < 0$ for the toy model and the probability S_2 to enter the throat in the D1-D5 geometry. On physical grounds we expect

$$P_1 = S_1^2 \tag{B.1}$$
$$P_2 = S_2^2 \tag{B.2}$$

and we will verify that we indeed obtain these relations. In particular S_2 will be the probability for an incident quantum to enter the extremal D1-D5 throat.

We begin with the toy model defined in the subsection 4.1. Using the relation

$$B = D\left(1 + \frac{ic}{2k}(1 - e^{2ikL})\right), \tag{B.3}$$

we find that the wave which looks like

$$\Phi_k(x,t) = B\left[e^{-ikt-ikx} + \frac{A}{B}e^{-ikt+ikx}\right] \tag{B.4}$$

for positive x, in the inner region becomes:

$$\begin{aligned} \Phi_k(x,t) &= -iB\frac{2k}{c}\frac{1}{1-\frac{2ik}{c}-e^{2ikL}}\left[e^{-ikt-ikx} - e^{2ikL}e^{-ikt+ikx}\right] \\ &= -iB\frac{2k}{c}\frac{1}{1-\frac{2ik}{c}}\sum_{n=0}^{\infty}\left(\frac{e^{2ikL}}{1-\frac{2ik}{c}}\right)^n\left[e^{-ikt-ikx} - e^{2ikL}e^{-ikt+ikx}\right] \end{aligned} \tag{B.5}$$

Thus at the lowest order in $\frac{2k}{c}$ the probability for going beyond $x = 0$ is

$$S_1 = \left(\frac{2k}{c}\right)^2 \tag{B.6}$$

Comparing this result with (4.15) we find that $P_1 = S_1^2$ as we could anticipate from the beginning.

Let us now look at absorption by the rotating D1–D5 system. As in the case of the toy model, we need to evaluate the ratio of coefficients in front of the left movers in the inner and outer region:

$$R' = \frac{D_1 e^{i\pi\epsilon'/2} - D_2(-1)^l e^{-i\pi\epsilon'/2}}{C_1 e^{i\pi\epsilon/2} - C_2(-1)^l e^{-i\pi\epsilon/2}} \tag{B.7}$$

We will ignore the case of resonance frequencies, then in the leading order in ϵ and ϵ' we will replace $e^{\pm i\pi\epsilon/2}$ and $e^{\pm i\pi\epsilon/2}$ by 1. Then denominator of (B.7) becomes $C_1 - C_2(-1)^l$ and after substituting this expression from (3.36), we get

$$R' = \left[D_1 - D_2(-1)^l\right]\frac{D_1 - D_2(-1)^l}{D_2 C_2}\frac{1}{\epsilon\epsilon'}\left(\frac{Q_1Q_5\omega^4}{16R^4}\right)^{l+1}\left[\frac{1}{(l+1)!l!}\right]^2. \tag{B.8}$$

Taking into account the value of C_2 (3.37), we rewrite this expression as

$$R' = -\frac{D_1 - D_2(-1)^l}{D_2(-1)^l}\frac{1}{\epsilon'}\left(\frac{Q_1Q_5\omega^4}{16R^4}\right)^{\frac{l+1}{2}}\left[\frac{1}{(l+1)!l!}\right]. \tag{B.9}$$

Substituting here the value of the ratio (3.33), we finally get

$$R' = \pi i\frac{1 + e^{2\pi i(\beta-\alpha+l/2)}}{1 - e^{2\pi i(\beta-\alpha+l/2)}}\left(\frac{Q_1Q_5\omega^4}{16R^4}\right)^{\frac{l+1}{2}}\left[\frac{1}{(l+1)!l!}\right]. \tag{B.10}$$

Making a formal expansion of the denominator we get

$$R' = \pi i \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{\frac{l+1}{2}} \left[\frac{1}{(l+1)!l!}\right] \left\{1 + 2\sum_{n=1}^{\infty} e^{2\pi i n(\beta - \alpha + l/2)}\right\} \tag{B.11}$$

Then the leading contribution to the probability P_2 is

$$S_2 = 4\pi^2 \left(\frac{Q_1 Q_5 \omega^4}{16R^4}\right)^{l+1} \left[\frac{1}{(l+1)!l!}\right]^2 \tag{B.12}$$

We again note that $P_2 = S_2^2$, which was anticipated. The probability P_2 of going in the throat coincides with absorption coefficient a_l for the unexcited D1-D5 geometry, which was evaluated in [4].

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